Quasistationary distributions of dissipative nonlinear quantum oscillators in strong periodic driving fields

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The dynamics of periodically driven quantum systems coupled to a thermal environment is investigated. The interaction of the system with the external coherent driving field is taken into account exactly by making use of the Floquet picture. Treating the coupling to the environment within the Born-Markov approximation one finds a Pauli-type master equation for the diagonal elements of the reduced density matrix in the Floquet representation. The stationary solution of the latter yields a quasistationary, time-periodic density matrix which describes the long-time behavior of the system. Taking the example of a periodically driven particle in a box, the stationary solution is determined numerically for a wide range of driving amplitudes and temperatures. It is found that the quasistationary distribution differs substantially from a Boltzmann-type distribution at the temperature of the environment. For large driving fields it exhibits a plateau region describing a nearly constant population of a certain number of Floquet states. This number of Floquet states turns out to be nearly independent of the temperature. The plateau region is sharply separated from an exponential tail of the stationary distribution which expresses a canonical Boltzmann-type distribution over the mean energies of the Floquet states. These results are explained in terms of the structure of the matrix of transition rates for the dissipative quantum system. Investigating the corresponding classical, nonlinear Hamiltonian system, one finds that in the semiclassical range essential features of the quasistationary distribution can be understood from the structure of the underlying classical phase space.

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I. INTRODUCTION

Within the Born-Markov approximation, autonomous open quantum systems are described by quantum dynamical semigroups with a time-independent Lindblad generator [1]. Under quite general physical conditions such systems relax in the long-time limit to a unique stationary state, which is given by the principles of equilibrium statistical mechanics [2]. For example, requiring the condition of detailed balance for the transition rates and some kind of ergodic property regarding the operators which describe the coupling of the system to its environment, one finds an equilibrium stationary state which is given by the Boltzmann distribution over the energy eigenvalues of the system.

For open quantum systems in time-varying external fields, the quantum dynamics must be described, in general, by a time-dependent generator. In the case in which the external driving field is strong, one expects that the long-time dynamics differs significantly from the equilibrium stationary state. In this paper, we shall investigate the question of the existence and the basic properties of a certain quasistationary state which governs the long-time behavior for systems in strong, time-periodic driving fields.

In our study, the interaction with the external field will be treated exactly using the Floquet representation for time-periodic quantum systems [3,4], whereas the coupling to environment will be taken into account in the Born-Markov approximation. It is known that for this case the diagonal elements of the reduced density matrix in the Floquet representation obey a closed equation of motion which is formally equivalent to a Pauli-type master equation [5–7].

We shall perform analytical and numerical investigations of the stationary solution of the Pauli master equation for a general class of periodically driven, nonlinear oscillators coupled to an environment at finite temperature. Our results reveal that a large class of Hamiltonian systems leads to a unique, quasistationary density matrix which is diagonal in the Floquet representation. The structure of the quasistationary distribution will be discussed in detail. We shall also study the connections to the phase flow of the corresponding classical Hamiltonian system, which shows a sharp dichotomy of quasiregular and chaotic motion.

II. MASTER EQUATION FOR OPEN QUANTUM SYSTEMS IN STRONG DRIVING FIELDS

A. The density matrix in the Floquet representation

We consider in the following a periodically driven quantum system coupled to an environment at temperature T (for a review, see Ref. [7]). The coherent part of the dynamics is generated by some Hamiltonian $H(t)$ which is periodic in time with frequency ω_L, that is, we have $H(t+T_L)=H(t)$, where $T_L=2\pi/\omega_L$ denotes the period. Usually, $H(t)$ takes the form $H(t)=H_0+H_I(t)$, where H_0 is the unperturbed system Hamiltonian and $H_I(t)$ represents a time-periodic interaction with an external driving field.

According to the Floquet theorem [3,4], there exists a basis of T_L-periodic wave functions $u_j(t)=u_j(t+T_L)$, the Floquet states, such that any solution $\psi(t)$ of the time-dependent Schrödinger equation pertaining to the Hamiltonian $H(t)$ can be represented in the form (we choose units such that $\hbar=1$)

$$\psi(t)=\sum_j a_j e^{-i\varepsilon_j t}u_j(t).$$

The quantity ε_j is the Floquet index or quasienergy corresponding to the Floquet state $u_j(t)$. The important point to note is that the amplitudes a_j in the above representation are independent of time.

In principle, the coupling to the environment may take any form. To be specific, we take this interaction to be of dipole form, that is, the relevant system operator which couples to the degrees of the environment is the dipole operator D. Such a coupling occurs, for example, in atomic or quantum optical systems where the environment could be the quantized radiation field in thermal equilibrium [1,8]. Invoking the Floquet representation, one finds that the dynamics of the diagonal matrix elements

$$p_j(t)\equiv\langle u_j(t)|\rho(t)|u_j(t)\rangle \tag{1}$$

of the reduced density operator $\rho(t)$ of the open system is governed by the following Pauli master equation in the strong driving limit:

$$\frac{d}{dt}p_j(t)=\sum_k \{\mathbf{w}_{jk}p_k(t)-\mathbf{w}_{kj}p_j(t)\}\equiv\sum_k \mathbf{W}_{jk}p_k(t). \tag{2}$$

This equation may be derived directly within the density-matrix formulation [5] or else by making use of the stochastic wave-function method and by investigating the associated jump process [6]. In both cases, one uses the Born-Markov approximation for the coupling of the reduced system to its environment, whereas the coupling to the driving field is treated exactly by invoking the Floquet representation of the time-evolution operator.

Formally, Eq. (2) represents an ordinary master equation in the sense of classical probability theory for a stochastic jump process [9]. The quantity $\mathbf{w}_{kj}$ is the rate (probability per unit of time) for a jump from the Floquet state $u_j(t)$ into the Floquet state $u_k(t)$. For dipole coupling, these rates are given by the explicit expression [6]

$$\mathbf{w}_{kj}=\sum_m \mathbf{w}_{kjm}=\sum_m \gamma(\omega_{kjm})\overline{N}(\omega_{kjm})|D_{kjm}|^2. \tag{3}$$

Here, the sum is to be extended over all integers m which label the Fourier modes D_{kjm} of the time-periodic dipole matrix element $\langle u_k(t)|D|u_j(t)\rangle$,

$$D_{kjm}=\int_0^{T_L}\frac{dt}{T_L}\,e^{-im\omega_L t}\langle u_k(t)|D|u_j(t)\rangle. \tag{4}$$

The ω_{kjm} denote the corresponding transition frequencies which are given though differences of quasienergies plus integer multiples of ω_L,

$$\omega_{kjm}=\varepsilon_k-\varepsilon_j+m\omega_L. \tag{5}$$

Finally, $\gamma(\omega)=\gamma(-\omega)$ denotes the density of modes of the environment belonging to the frequency ω. For $\omega>0$, $\overline{N}(\omega)$ is the Planck distribution for the quanta of frequency ω. For the sake of a compact notation we define

$$\overline{N}(\omega)=\begin{cases}(e^{\beta\omega}-1)^{-1} & \text{for}\quad \omega>0,\\ \overline{N}(-\omega)+1, & \text{for}\quad \omega<0,\end{cases}$$

where $\beta=1/k_BT$, T is the temperature of the environment, and k_B denotes the Boltzmann constant.

B. The quasistationary solution

Any master equation of the form (2) has at least one stationary solution p_j^* [9]. This is due to the fact that the matrix $\mathbf{W}_{jk}$ has always a left eigenvector $(1,1,1,\dots)$ belonging to the eigenvalue zero. The corresponding right eigenvector p^* then fulfills $\mathbf{W}p^*=0$, and, when normalized, is a stationary solution of the master equation.

Once we have determined a stationary distribution p_j^* of the Pauli-type master equation (2), we immediately obtain a solution $\rho^*(t)$ of the corresponding density-matrix equation which is diagonal in the Floquet representation and which is given in the Schrödinger picture by

$$\rho^*(t)=\sum_j |u_j(t)\rangle p_j^*\langle u_j(t)|. \tag{6}$$

This equation represents a density matrix which varies periodically in time with a period which is equal to that of the external driving field. For this reason the stationary solution p_j^* of the Pauli-type master equation may be called *quasistationary*.

An important question is whether the diagonal part of any initial density matrix converges for large times to the quasistationary solution. For this to be the case, the stationary solution p_j^* of the Pauli master equation must be unique, which means that the Pauli master equation must be irreducible [9]. A similar condition is used in the study of the return to equilibrium in relaxing semigroups of autonomous open quantum systems [2].

For general rates $\mathbf{w}_{kj}$ the determination of the quasistationary solution p_j^* can be a difficult task. However, for an autonomous physical system one expects that any initial state relaxes to a state which is in thermal equilibrium with the environment. Thus, without external driving, the stationary solution of Eq. (2) should represent a canonical distribution over the energy eigenvalues ε_j^0 of the unperturbed system Hamiltonian H_0. It is instructive for the considerations below to recall briefly the basic arguments which lead to this conclusion. For an autonomous system without external driving field, the master equation is of the same form as Eq. (2), where, however, the transition rates are given by

$$\mathbf{w}_{kj}^0=\gamma(\omega_{kj})\overline{N}(\omega_{kj})|D_{kj}|^2.$$

The transition frequencies are now obtained as differences of unperturbed energy eigenvalues, $\omega_{kj}=\varepsilon_k^0-\varepsilon_j^0$, and $D_{kj}=\langle\varphi_k|D|\varphi_j\rangle$ is the dipole matrix element between the corresponding eigenstates φ_k and φ_j of H_0. According to general principles of statistical mechanics [9], the stationary equilibrium distribution p_j^* of a closed physical system obeys the condition of detailed balance which is given by $\mathbf{w}_{kj}^0p_j^*=\mathbf{w}_{jk}^0p_k^*$. The crucial point is that the rates $\mathbf{w}_{kj}^0$ for the autonomous system do not involve a sum over the index m

which labels the Fourier components in Eq. (3). Therefore, the density of the modes of the environment as well as the dipole matrix element drop out when one forms the ratio

$$\frac{\mathbf{w}_{kj}^0}{\mathbf{w}_{jk}^0}=\frac{\overline{N}(\omega_{kj})}{\overline{N}(\omega_{kj})+1}=e^{-\beta\omega_{kj}}, \tag{7}$$

where we assume (without restriction) that $\omega_{kj}>0$. As can be seen immediately, the solution p_j^* of the detailed balance condition yields, as expected, the canonical distribution $p_j^* = \mathcal{N}\exp(-\beta\varepsilon_j^0)$, where $\mathcal{N}$ is a normalization factor. This is the usual argument employed in statistical mechanics, demonstrating that the system relaxes to a stationary equilibrium state which is given by the canonical distribution over the unperturbed energy eigenvalues at the given temperature T of the environment.

It is interesting to observe that a similar argument applies also to another case, namely the periodically driven harmonic oscillator with system Hamiltonian,

$$H(t)=\frac{1}{2\mu}p^2+\frac{1}{2}\mu\omega_0^2x^2+\lambda x\sin\omega_L t.$$

In this case one obtains [6]

$$D_{kjm}=\delta_{m,0}(\delta_{j+1,k}\sqrt{j+1}+\delta_{j-1,k}\sqrt{j})\sqrt{\frac{1}{2\mu\omega_0}}.$$

This expression shows that for $|j-k|\neq 1$ the rates $\mathbf{w}_{kj}$ vanish. For $|j-k|=1$, however, only the term $m=0$ in Eq. (3) is different from zero. This is a very specific property which is valid only for harmonic potentials. Thus, the dipole operator couples only neighboring Floquet states $u_j(t)$ and $u_{j\pm 1}(t)$ and the expression for the rates $\mathbf{w}_{kj}$ involves only a single Fourier component, as is the case for an autonomous system. On using the same arguments as above, we therefore get the following quasistationary distribution:

$$p_j^*=\mathcal{N}e^{-\beta\varepsilon_j}, \tag{8}$$

where ε_j is the quasienergy spectrum of the harmonic oscillator (see, e.g., [10]). Note that the quasienergies of the driven harmonic oscillator differ from the unperturbed energies just by a j-independent term. The quasienergies of the harmonic oscillator are thus equidistant for all λ. Equation (8) implies that the stationary, nonequilibrium distribution of the driven oscillator is a canonical distribution over its quasienergy states, that is, the quasistationary density matrix $\rho^*(t)$ varies periodically in time with time-independent occupation probabilities of the Floquet states. A more detailed investigation of the dynamics of dissipative, periodically driven systems with quadratic potentials can be found in Refs. [11,12].

We now turn to the case of a driven, nonlinear oscillator. Instead of the simple relation (7), we have, in general, the following expression for the ratio of transition rates:

$$\frac{\mathbf{w}_{kj}}{\mathbf{w}_{jk}}=\frac{\sum_m \mathbf{w}_{kjm}}{\sum_m \mathbf{w}_{jkm}}=\frac{\sum_m \mathbf{w}_{kjm}}{\sum_m \mathbf{w}_{jk,-m}}$$
$$=\frac{\sum_m \gamma(\omega_{kjm})\overline{N}(\omega_{kjm})|D_{kjm}|^2}{\sum_m \gamma(\omega_{kjm})\overline{N}(-\omega_{kjm})|D_{kjm}|^2}, \tag{9}$$

where we have used the relations $\omega_{jk,-m}=-\omega_{kj,+m}$, $\gamma(-\omega)=\gamma(\omega)$, and $D_{jk,-m}=D_{kj,+m}^*$, which are valid by definition.

In the case of strong driving, many Fourier modes of the Floquet wave functions are excited with appreciable amplitude. This means that $|D_{kjm}|^2$ may be appreciably different from zero for many m's which label these modes. Equation (9) therefore shows that the ratio $\mathbf{w}_{kj}/\mathbf{w}_{jk}$ of transition rates differs significantly from the simple relation (7), which is valid in the zero driving limit. Therefore, the simple line of reasoning leading to a stationary state given by a canonical distribution does not apply in the present case. In the next section we shall investigate numerically the properties of the quasistationary distribution p_j^* of the Pauli-type master equation for the case of a nonlinear, strongly driven oscillator.

III. NUMERICAL SIMULATIONS

A. Model system and numerical methods

As an example for a strongly anharmonic system, we consider a periodically driven particle in a potential box. The time-dependent Hamiltonian is given by

$$H(t)=-\frac{1}{2\mu}\frac{d^2}{dx^2}+V(x)+\lambda x\sin\omega_L t, \tag{10}$$

where the potential $V(x)$ reads

$$V(x)=\begin{cases}0 & \text{for} \quad |x|<a,\\ +\infty & \text{for} \quad |x|>a.\end{cases} \tag{11}$$

Scaling space, time, and momentum coordinates as

$$\hat{x}=\frac{x}{a},\quad \hat{t}=\omega_L t,\quad \hat{p}=\frac{p}{\mu a\omega_L},$$

the Schrödinger equation corresponding to the Hamiltonian (10) can be written as

$$i\frac{1}{\alpha}\frac{d}{d\hat{t}}\psi=\left\{-\frac{1}{2\alpha^2}\frac{d^2}{d\hat{x}^2}+\hat{V}(\hat{x})+\beta\hat{x}\sin\hat{t}\right\},$$

where $\hat{V}(\hat{x})=V(a\hat{x})/\mu a^2\omega_L^2$ is the scaled potential and we have introduced the dimensionless parameters

$$\alpha=\mu a^2\omega_L,\quad \beta=\frac{\lambda}{\mu a\omega_L^2}. \tag{12}$$

The coherent part of the dynamics thus depends only upon the two dimensionless parameters α and β. The parameter β is a dimensionless coupling constant which is proportional to the field amplitude. The meaning of the parameter α may be seen by looking at the commutator of space and momentum coordinate, $[\hat{p},\hat{x}]=-i\alpha^{-1}$. Recall that we have set $\hbar=1$. The quantity α^{-1} is thus a dimensionless, scaled Planck constant. The limit $\alpha\rightarrow\infty$ corresponds to the classical limit, whereas small values of α imply that quantum behavior dominates in a certain region of the classical phase space [10].

The dissipative part of the dynamics introduces two further parameters, namely the density of modes and the temperature of the environment. For simplicity we chose a constant density of modes, $\gamma(\omega)=\gamma_0$. γ_0 determines the relaxation time of the process but not the stationary solution. The latter only depends on the dimensionless temperature

$$\hat{T}=\frac{k_B T}{\omega_L}. \tag{13}$$

Thus, the stationary solution p_j^* depends on three dimensionless parameters, namely α, β, and $\hat{T}$.

The numerical determination of the stationary distribution p_j^* over the Floquet states proceeds in three steps as follows.

(i) Determination of the quasienergies and Floquet wave functions pertaining to the Hamiltonian (10).

(ii) Calculation of the Fourier components D_{kjm} of the dipole operator and determination of the matrix $\mathbf{W}$ of the master equation (2).

(iii) Determination of the (normalized) right eigenvector p_j^* of $\mathbf{W}$ corresponding to the eigenvector zero.

To perform step (i) we have represented the time-dependent Schrödinger equation in a finite basis consisting of N eigenfunctions of H_0. The Floquet spectrum is determined by diagonalization of the monodromy operator $U(T_L,0)$, that is, the time-evolution operator $U(t,t_0)$ of the time-dependent Schrödinger equation taken over a period of the driving field. With the help of the Floquet states, one evaluates the time-dependent matrix elements $\langle u_j(t)|D|u_k(t)\rangle$ of the dipole operator.

In step (ii) one determines the Fourier transform of the dipole matrix elements to obtain the Fourier components D_{kjm}. We denote by $m_{\max}$ the number of sampling points which are used used in the numerical Fourier tranformation. The Fourier components of the dipole matrix elements together with the quasienergies determined in step (i) yield the rates W_{kj} and the matrix $\mathbf{W}$.

In step (iii) one has to find the zero mode of the matrix $\mathbf{W}$. In all cases considered the lowest eigenvalue of $\mathbf{W}$ turned out to be smaller than the other eigenvalues by a factor of at least 10^4. This clearly excludes the possibility of a degenerate zero mode, and of a decomposable $\mathbf{W}$ matrix [9]. In all cases we thus have a unique stationary solution p_j^*.

B. Numerical results and discussion

In the following we shall represent the stationary distribution p_j^* as a function of the mean energies $\overline{E_j}$ which are obtained by averaging the expectation value of the time-dependent Hamiltonian $H(t)$ in the Floquet states $u_j(t)$ over a period of the external field,

$$\overline{E_j}=\int_0^{T_L}\frac{dt}{T_L}\langle u_j(t)|H(t)|u_j(t)\rangle. \tag{14}$$

We display in Fig. 1 a logarithmic plot of the stationary distributions p_j^* for a fixed driving amplitude of $\beta=0.248$ and for different scaled temperatures $\hat{T}=0.5,1.0,1.5,\ldots,8.0$. As can be seen from the figure, the canonical distribution over the unperturbed energies changes significantly for strong driving fields. The most striking feature is that the stationary distribution p_j^* exhibits two qualitatively very different and clearly separated regions. In the first region we have a number of states which are populated with an approximately constant probability. This region will be called plateau region in the following. For increasing mean energy the plateau region goes over to a second region where p_j^* clearly decays exponentially with the mean energy. A similar conclusion was found in an investigation of chaotic tunneling in a double-well potential [13].

Figure 1 also demonstrates that the plateau region is separated from the exponential region by a sharp transition at some mean energy $\overline{E_C}$ which is nearly independent of the temperature. The solid lines of Fig. 1 are obtained by a linear fit of the numerical data in the exponential region. One finds that the slopes of these lines are in perfect agreement with the chosen scaled temperatures $\hat{T}$ of the environment. In the

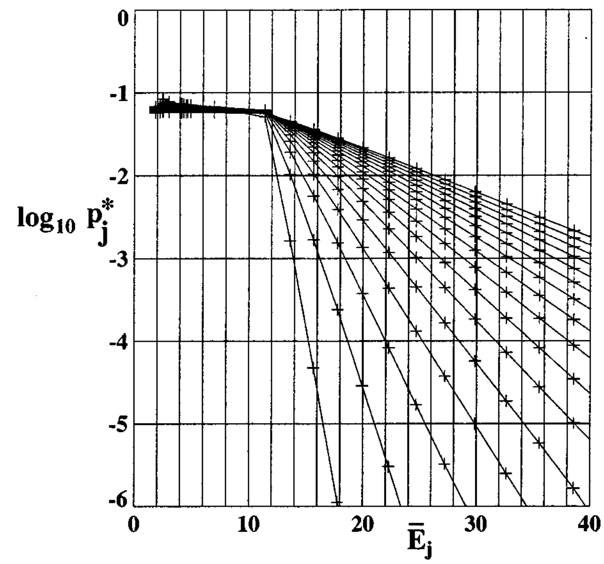

FIG. 1. Logarithmic plot of the quasistationary distribution p_j^* for the open system with Hamiltonian (10) as a function of the mean energy $\overline{E_j}$ for different scaled temperatures $\hat{T}=0.5,1.0,1.5,\ldots,8.0$ (symbols). The lines represent a linear fit of the numerical data in the exponential region, which is found to be in excellent agreement with a Boltzmann-type distribution corresponding to the various temperatures. The plateau region as well as the temperature independence of the mean transition energy $\overline{E_C}$ which separates both regions are also clearly seen. The parameters are $\alpha=20$, $\beta=0.248$, $N=32$, and $m_{\max}=2048$.

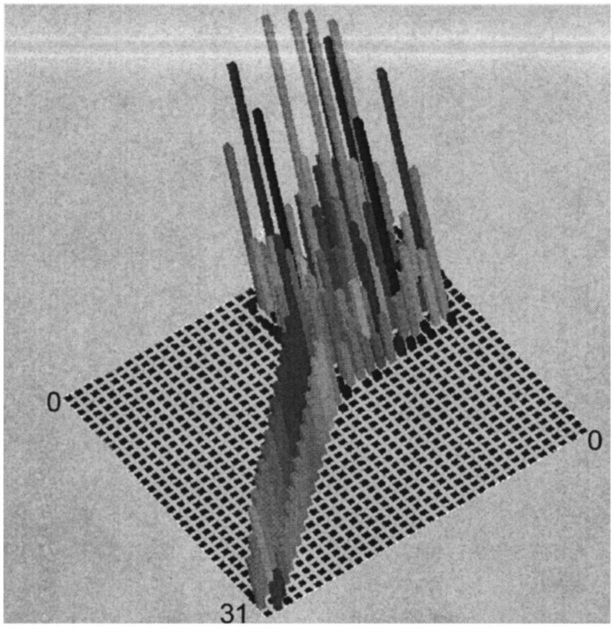

FIG. 2. Representation of the $(N\times N)$-matrix $\mathbf{w}_{kj}$ formed by the various rates for the transition between the Floquet states for the Hamiltonian (10). The matrix element $\mathbf{w}_{00}$ is found in the upper corner. The parameters are $\alpha=20$, $\beta=0.418$, $\hat{T}=4.5$, $N=32$, and $m_{\max}=2048$.

exponential region the stationary distribution therefore represents a canonical distribution over the mean energies of the Floquet states.

Summarizing these results, we may write for the stationary distribution in the plateau region

$$p_j^* \approx \text{const} \quad \text{for} \;\; \overline{E}_j < \overline{E}_C, \tag{15}$$

and in the exponential region

$$p_j^* \propto \exp\{-\overline{E}_j/k_BT\} \quad \text{for} \;\; \overline{E}_j > \overline{E}_C. \tag{16}$$

In order to explain this behavior of the quasistationary distribution, we plot in Fig. 2 the matrix $\mathbf{w}_{kj}$. As can be seen from the figure, for the chosen parameters the states $j=1,\ldots,15$ are strongly mixed: Each Floquet state is coupled to all other Floquet states from this set and the corresponding transition rates vary erratically with k and j. This explains why in the plateau region the stationary solution is nearly constant since all rows and columns of the $\mathbf{W}$ matrix sum up to nearly the same value in this range.

However, above a certain sharp threshold, which is given by $j=15$ for the parameters of the figure, the transition rates strongly decrease with increasing j and k and couple only states with $|j-k|=1$. This behavior is very similar to that of the harmonic oscillator and explains why for $j>15$ the stationary solution p_j^* behaves in a way which is similar to that of the distribution of the harmonic oscillator.

C. Comparison with the classical phase-space structure

The appearance of two clearly separated regions in the stationary distribution p_j^* can also be understood from a simple semiclassical analysis. To this end, we first investigate the classical analog of the quantum system given by the Hamiltonian (10). The classical phase flow generated by the corresponding Hamiltonian function is known to be chaotic and has been extensively studied in the literature (see, e.g., [14,15]).

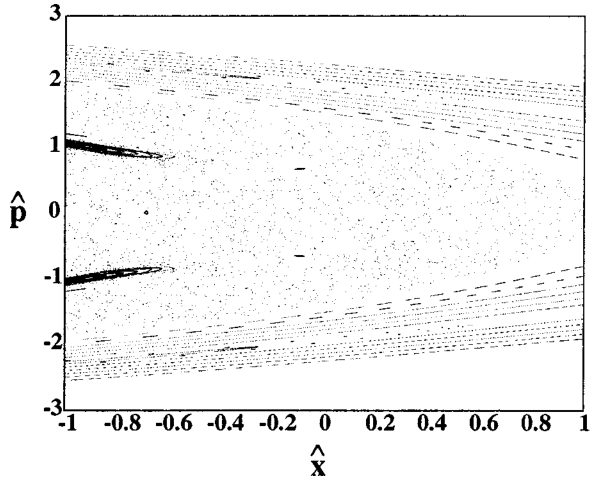

FIG. 3. Poincaré surface of section for the strongly driven particle in the box for a scaled driving amplitude of $\beta=0.66$. The figure represents a family of solutions of the classical equations of motion in the $(\hat{x},\hat{p})$ plane at times $t=0,T_L,2T_L,\ldots,600T_L$, where T_L denotes the period of the driving field.

For strong driving fields, that is, for driving amplitudes $\beta>\beta_c$ which are larger than the amplitude $\beta_c=0.0625$ given by the Chirikov criterion [16] for the overlap of all primary resonances, the classical phase space also consists of two clearly separated regions [15]. This can be seen from Fig. 3 which shows a Poincaré surface of section of the phase space.

The first region constitutes a connected chaotic sea which emerges from the region of the primary nonlinear resonances and which contains only small stable, elliptic islands. The second phase-space region is densely filled with invariant tori corresponding to perpetual adiabatic invariants [15]. We see from Fig. 3 that both regions are separated by a sharp border which marks the transition from the irregular, chaotic motion to the quasiperiodic, nearly integrable motion in the region surrounding the chaotic sea.

As is shown in [10], the quantization of the invariant flux tubes in the regular region of the phase space yields an excellent semiclassical approximation for the Floquet states and the quasienergies of the quantum system. In view of our above results on the quasistationary probability distribution of the open, dissipative system, it is tempting to relate the exponential region of that distribution to the nearly integrable region of the phase space. In fact, for $\overline{E}_j>\overline{E}_C$ the quasienergies rapidly approach the mean energies $\overline{E}_j$ and one expects a Boltzmann distribution over the quasienergies in this region of phase space.

On the other hand, the quasienergy spectrum for the states corresponding to the chaotic sea shows a complicated avoiding crossing structure when plotted as a function of the driving amplitude. This reflects large dipole matrix elements and a broad Fourier spectrum of these elements. Consequently, one expects that the chaotic sea corresponds to the plateau region observed in the stationary distribution p_j^*.

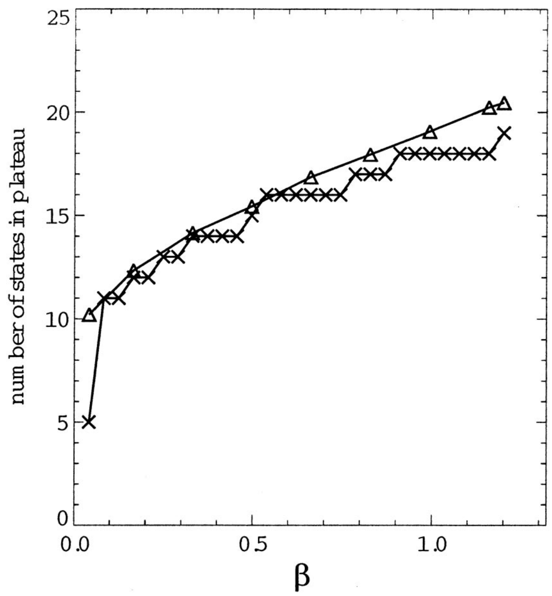

FIG. 4. The number of Floquet states in the plateau region of the quasistationary distribution p_j^* as a function of the scaled driving amplitude β for fixed $\alpha=20$. The figure shows this number as it is obtained from the full quantum calculation (crosses) and compares it with the number of semiclassical Floquet states corresponding to the chaotic sea of the classical phase space (triangles) estimated by means of Eq. (17).

To verify this simple semiclassical picture, we study the behavior of the quasistationary distribution as a function of the scaled driving amplitude β. Figure 4 shows the number of Floquet states in the plateau region of p_j^* determined from the full quantum calculation (crosses) and compares it with the number of semiclassical Floquet states corresponding to the chaotic sea of the classical phase space (triangles). The number of semiclassical Floquet states corresponding to the chaotic sea is estimated as follows. First, we have determined from the Poincaré surface of section the area A of the chaotic sea in the scaled coordinates $\hat{x}$, $\hat{p}$. According to the quantization rules derived in [10], we then get for the number n of semiclassical Floquet states in that region the estimate

$$n=\frac{\alpha}{2\pi}A. \tag{17}$$

This relation simply expresses Weyl's rule applied to the extended phase space of the Hamiltonian system. It has already been used in Ref. [17] for an investigation of the mixed regular and chaotic dynamics of a driven rotor. Note that A and n depend on β. It is this function $n=n(\beta)$ which is represented in Fig. 4 (triangles). The agreement between both quantities plotted in this figure nicely confirms our above semiclassical picture.

IV. SUMMARY

We have studied the dynamics of open quantum systems subjected to strong, periodic driving fields. The stationary solution of the Pauli-type master equation which governs the diagonal elements of the reduced density matrix in the Floquet representation has been demonstrated to differ substantially from a canonical distribution at the temperature of the environment: It exhibits a certain region in which a number of Floquet states is occupied with approximately constant probability. This plateau region is clearly separated from an exponential tail of the stationary distribution which describes a Boltzmann-type distribution at the environmental temperature over the mean energies of the Floquet states. The number of Floquet states within the plateau region is nearly independent of the temperature but strongly depends on the driving amplitude.

The essential features of the stationary solution can be understood from an investigation of the classical phase-space structure. It has been shown that the plateau region corresponds to the chaotic sea which emerges from the region of phase space belonging to the primary nonlinear resonances. This chaotic sea is surrounded by a nearly integrable phase-space region which is densely filled with invariant flux tubes. This region which is dominated by regular motion corresponds to the exponential tail of the stationary distribution. A sharp transition border separates the chaotic sea from the nearly integrable motion and marks the transition from the plateau region of the stationary distribution to its exponential tail.

These results have been obtained from numerical simulations of a simple, strongly nonlinear model, namely from the periodically driven particle in a potential box. It must be emphasized, however, that the dichotomy of the classical phase space as well as the general structure of the matrix $\mathtt{w}_{kj}$ describing the transition rates between Floquet states is similar for all potentials that lead to a discrete spectrum of unperturbed energy eigenvalues whose spacing increases with increasing energy. For strong driving fields our results thus describe generic features for this class of potentials.

We remark finally that our formulation of the problem of periodically driven open systems also allows the determination of the quanta radiated during the jumps between Floquet states. The frequencies of these quanta are determined by relation (5). The above properties of the quasistationary state of the open system could thus lead to characteristic features of the radiation spectrum.

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